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Discussion of "Copulas: Tales and facts", by Thomas Mikosch

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The contribution of Thomas Mikosch to the copula debate or indeed frenzy is to be welcomed. Many of the arguments presented no doubt need explicit mentioning, especially the more advanced theory of multivariate extremes holds promise for applications to quantitative risk management. For an extra reference to this theory, see Balkema and Embrechts (2006). Also the queries on two-stage versus full multivariate models and on truly dynamic models within the general, well established theory of stochastic processes are pertinent. Rather than taking up some of the technical issues, I will concentrate in my discussion on some historical and pedagogical facts related to copula modelling. Through the influential paper of Embrechts et al. (2002) we were partly responsible for the Google increase from 10,000–650,000 and no doubt by now well beyond that.

We wrote the paper [6] first as a RiskLab report in 1998 as a pedagogically oriented contribution warning against an over-optimistic view that correlation coefficients suffice for describing dependence, especially in the presence of extremes. We had already seen several examples in industry where this view led to big losses and erroneous results. In real financial practice, whether we like it or not, quantitative risk management questions often come to use in two stages: we add dependence considerations to marginal models, and that is exactly what (in a static, one-period) framework the copula (or Sklar) representation

$$P\left(X_1 \leq y_1, \ldots, X_d \leq y_d\right) = C\left(F_1\left(y_1\right), \ldots, F_d\left(y_d\right)\right) \tag{1}$$

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entails. Situations where this happens naturally are for instance multi-line products in non-life insurance, aggregation models across different risk types in finance, the AMA modelling of operational risk under Basel II. Often in these, and other situations, there is no hope to obtain at the moment a global, dynamic multivariate model. The two-stage approach, using Eq. 1 from right to left, allows to come up with a range of models/answers which help the user to understand the model risk present. For examples of such results, see Section 6.2 in McNeil et al. (2005). Neslehova et al. (2006) contain an application to operational risk data from the Basel II Third Quantitative Impact Study (QIS3). These, and many more examples, use the (admittedly trivial) representation Eq. 1 to warn that the data information available (like in the QIS3 case) does not allow for a risk measure (i.e., VaR) to be estimated precisely. I personally have always stated that there are three reasons why copulas are important: pedagogic, pedagogic and stress testing. The twice pedagogic goes back to the mid-1990s where the limitations of correlation driven multivariate models for risk management purposes were (and perhaps still are) poorly understood. The stress testing (which is mandatory both under Basel II as well as Solvency two) is to be interpreted as explained above: given that we just have marginal risk information and some vague idea of dependence, what range of possibilities exist for a risk measure of the global position. The notion of copula has been extremely useful in understanding the limitations caused by the data non-availability in financial and insurance risk management. That any "new" risk management tool can (and will) be misused beyond its original intent is clear and well-known from many other cases. If the word "copula" has contributed for many applied researchers to think more carefully about multivariate modelling, then that is a success. If the same researchers have looked more in detail at some of the classical publications and texts on the subject, like Joe (1997) and Schwiezer and Sklar (1983) to name just two, then that is a bonus. It is up to us mathematicians to also point at the (for us obvious) limitations. The sentence "... that some include the word "copula" in the title of their paper not because they contribute to the theory of copulas, but because they believe that one can publish easier" may have been true early on but by now is definitely not the case any more. Already at the ASTIN meeting in Berlin (2003) Gunnar Benktander turned to me saying: "there are too many talks on copulas." Fundamental research on copulas has been going on for more than fifty years and will continue, interesting applications will no doubt continue to be found, less interesting papers misusing the concept will unfortunately also continue to be written. I personally hope that mathematical finance and insurance will turn to truly (high-dimensional) multivariate modelling in QRM, but at the same time gets concerned with some of the real issues out there; I recall Chris Rogers making such a call for duty already in 1998. If the paper by Thomas Mikosch turns out to be a constructive addition to reaching that goal, then it is to be highly welcomed.

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